ISSN (ONLINE) 2583-2506
http://publications.rayatbahrauniversity.edu.in
RBIJMR-Rayat Bahra International Journal of Multidisciplinary Research, Vol. 01, Issue 01, December 2021

A Neutrosophic Cross Entropy Measure for MADM Using Fuzzy Weighted Ideal Alternative vectors ☆

C.P.Gandhi[1]

University School Of Sciences, Rayat Bahra University, Chandigarh, India.[1,*]

Abstract

This study intends to establish a novel neutrosophic cross entropy based multi attribute decision making method, combined with some relative fuzzy weighted ideal alternative (RFWIA) vectors. RFWIA vectors have been introduced to get rid off the shortcomings of existing Archimedean, Einsteen and Hammer fuzzy weighted averaging aggregation(AFWA, EFWA, HFWA) operators where they may exhibit some ambiguity under some mathematical treatments. The applicability of the proposed symmetric single valued neutrosophic cross entropy measure is exemplified with some MADM problem on financial strategy. To obtain the optimal ranking orders of alternatives, the proposed neutrosophic cross entropy measure has been found remarkable in comparison with the enduring fuzzy cross entropy measures which may return either undefined or unreasonable ranking orders of alternatives in some situations.

Keywords: Fuzzy sets, symmetric fuzzy cross entropy,relative fuzzy weighted ideal alternative vector, fuzzy weighted averaging aggregation operators,
2010 MSC: 110.010

*Corresponding author : C.P. Gandhi
Email address: cchanderr@gmail.com (University School Of Sciences, Rayat Bahra University, Chandigarh, India.)

ISSN (ONLINE) 2583-2506
http://publications.rayatbahrauniversity.edu.in
RBIJMR-Rayat Bahra International Journal of Multidisciplinary Research, Vol. 01, Issue 01, December 2021

1. Introduction

In solving MADM problems, some of the biggest challenges that a DM has to face are - (i) how to solve MADM problems with fuzzy preference information on alternatives[4,5,18](ii) how to develop the effective methods for solving MADM analysis problems containing imprecise, undetermined and inconsistent data and (iii) how to rank the alternatives associated with conflicting and non-commensurate attributes[4,10,14].Due to time pressure and ambiguity in human thinking, the DM's preference information on alternatives may lead to vary his judgement skill in form and depth. The current methods for solving MADM problems with fuzzy preference information on alternatives include-((i) multi-dimensional scaling method with ideal point[13],(ii) interactice simple additive weighting method[10]and (iii) linear programming techniques for multi-dimensional analysis of preference[11,17] etc.However, these methods either need the ordering of preferences between pair of alternatives given by the DM or do not address the problem when the potential alternatives contain the preference information in terms of FSs. Zhi et all[23] proposed a linear goal programming model which can be used to construct the weight vector of the attributes and then to use it for selecting the most desirable alternatives.Wan et all[19] extended the concept of FSs to interval intuitionistic fuzzy sets for solving MADM problems with incomplete attribute weight information.Wei [20] developed an information measure based on picture fuzzy cross entropy for solving MADM problems and applied it for ranking different alternatives.Xu and yager[21] developed some geometric aggregation operators based on IFSs. Weighted aggregation(WA) operators have been used by many researchers for aggregating all the performance of the criteria for alternatives.The remainder of this paper is organized as under.

Section.2 addresses the concept of some preliminaries required for the subsequent development of the proposed k.Section.3 introduces a novel SFCE measure between two FSs for measu expected information based on the single degree of positive membership ɔwed by a new measure of fuzzy entropy. Section.4 presents some newly constructed RFWIA vectors based on fuzzy Archimedean, Einsteen and Hammer weighted averaging aggregation operators. Through the newly discovered SFWCE measure, the applicability and feasibility of the proposed method along with its comparative analysis with the existing method[23] based on linear goal programming is exemplified in Section.5.Finally, section.6 summarizes the concrete conclusions of the work done in this paper.

2. Preliminaries:-

The following section deals with the brief review of some basic concepts required for the development of the proposed work.

2.1. Fuzzy Set [22].Let $X = (x_1, x_2, ..., x_n)$ be a finite universe of discourse.Then a fuzzy set A is defined as $A = (< x, \mu_A(x) > |x\epsilon X)$ where $\mu_A(x) : X \to [0, 1]$ s.t. $0 \leq \mu_A(x) \leq 1$

2.2. A Measure of Fuzzy Entropy. Let $P(X)$ consists of all fuzzy subsets of X and $A\epsilon X$.Then $H(A) : P(X) \to R$ satisfies

$(i)H(A) \geq 0\forall\mu_A(x); 0 \leq \mu_A(x) \leq 1(ii)H(A) = 0$ iff $\mu_A(x) = 0$ or 1

$(iii)H(A^c) = H(A)$ (iv) It exhibits the concavity property each $\mu_A(x)$

(v) It has maximum value when $\mu_A(x) = \frac{1}{2}$ and

the maximum value increases for n.

2.3. A Measure of Fuzzy Cross Entropy. A cross entropy function $D : P(X) \times P(X) \to R$ satisfies

$(i)D(A, B) \geq 0(ii)D(A, B) = 0 \Leftrightarrow A = B(iii)D(A, F) =$ Max.H(A) $- H(A)$;

F is the most fuzzy vector (iv) It is a a convex function of both A and B

2.4.A SFCE Measure. This function $D : P(X) \times P(X) \to R$ satisfies

$(i)D(A, B) \geq 0(ii)D(A, B) = 0 \Leftrightarrow A = B(iii)D(A, B) = D(B, A)$

$(iv)D(A^c, B^c) = D(A, B)$ i.e, $D(A, B)$ remains same if B is replaced by A^c.

ISSN (ONLINE) 2583-2506
http://publications.rayatbahrauniversity.edu.in
RBIJMR-Rayat Bahra International Journal of Multidisciplinary Research, Vol. 01, Issue 01, December 2021

3. A Novel SFCE measure

To overcome the shortcomings of Bhandari and Pal[2]and Shang and Jiang[16] , an effort has been made to propose a novel SFCE measure as follows

Theorem.3.1 For any fuzzy set A in X, $H_F(A)$ represents a well defined fuzzy entropy measure where $H_F(A) = \sum_{i=1}^{n} \log_{\frac{3}{2}} \left[1 + \frac{1}{4}\sqrt{\mu_A(x_i)(1-\mu_A(x_i))}\right]$ Moreover, the minimum value of $H_F(A)$ is zero and maximum is $(\log_{\frac{3}{2}} \frac{9}{8})n$.

Proof The non-negativity of $H_F(A)$ is obvious since $0 \leq \mu_A(x) \leq 1$. It vanishes whenever $\mu_A(x)$ is zero or unity. Also, it does not change whenever $\mu_A(x)$ is replaced with its counter part $1 - \mu_A(x)$.

Concavity: Partial differentiation of $H_F(A)$ with respect to $\mu_A(x_i)$ yields

$$\frac{\partial H_F(A)}{\partial \mu_A(x_i)} = \frac{(1-2\mu_A(x_i))}{8\log\left[\frac{3}{2}\sqrt{\mu_A(x_i)(1-\mu_A(x_i))} + \frac{3}{8}\mu_A(x_i)(1-\mu_A(x_i)\right]}$$

$$\frac{\partial^2 H_F(A)}{\partial \mu^2_{A(x_i)}} = -\frac{1}{\log_{\frac{3}{2}}}\left[\frac{(1-2\mu_A(x_i))^2}{64\left[1+\frac{1}{4}\sqrt{\mu_A(x_i)-\mu_A^2(x_i)}\right]^2(\mu_A(x_i)-\mu_A^2(x_i))} + \frac{(1-2\mu_A(x_i))^2}{16\left[1+\frac{1}{4}\sqrt{\mu_A(x_i)-\mu_A^2(x_i)}\right][\mu_A(x_i)-\mu_A^2(x_i)]^{\frac{3}{2}}} + \frac{1}{4\left[1+\frac{1}{4}\sqrt{\mu_A(x_i)-\mu_A^2(x_i)}\right][\mu_A(x_i)-\mu_A^2(x_i)]^{\frac{1}{2}}}\right] \leq 0$$

for each $\mu_A(x_i)\epsilon[0,1]$. This establishes the fact that $H_F(A)$ is a concave function of each $\mu_A(x_i)$ and hence possess its maximum value.The concavity property of $H_F(A)$ can also be seen from its three dimensional plot as shown in Figure.1. For maximum value of $H_F(A)$, set $\frac{\partial H_F(A)}{\partial \mu_A(x_i)} = 0$ which implies $\mu_A(x_i) = \frac{1}{2}$.

$$\text{Also Max. } H_F(A) = H_F(A)|_{\mu_A(x_i)=\frac{1}{2}} = n\log_{\frac{3}{2}}\frac{9}{8} \qquad (1)$$

We now reinterporate the result of Theorem.3.1 intended to establish the proposed SFCE measure, the result of which will play a vital role for solving some MADM problem.

Theorem.3.2 Set $T_0 = \mu_A(x_i) + \mu_B(x_i), T_1 = \sqrt{\mu_A(x_i)\mu_B(x_i)}$ and $T_2 = \sqrt{(1-\mu_A(x_i))(1-\mu_B(x_i))}$, then $K_{FS}(A,B)$ is a well defined symmetric fuzzy

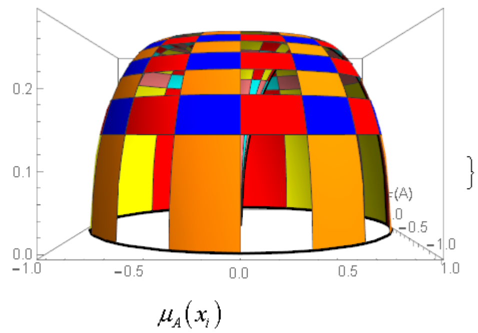

Figure 1: Concavity Property Exhibited by $H_F(A, B)$

cross entropy measure where

$$K_{FS}(A,B)=\sum_{i=1}^{n}\left[(2+T_0)\log_{\frac{3}{2}}\frac{1+\frac{T_0}{2}}{1+\frac{T_0+T_1}{3}}+(4-T_0)\log_{\frac{3}{2}}\frac{1+\frac{2-T_0}{2}}{1+\frac{2-T_0+T_2}{3}}\right]$$

Proof.$K_{FS}(A, B)$ meets the necessary conditions (i),(ii) and (iii) of Def.2.4. To establish the non-negativity of $K_{FS}(A, B)$, we first divert to prove the following lemma.

Lemma. Set $T_3 = \frac{\mu_A(x_i)+\sqrt{\mu_A(x_i)\mu_B(x_i)}+\mu_B(x_i)}{3} = \frac{T_0+T_1}{3}$, and $T_4 = \frac{\mu_A(x_i)+\mu_B(x_i)}{2} = \frac{T_0}{2}$. Then, $\exists$ the inequality:$T_3(\mu_A(x_i), \mu_B(x_i)) \le T_4(\mu_A(x_i), \mu_B(x_i)) \forall \mu_A(x_i), \mu_B(x_i) \epsilon [0, 1]$.

Proof. It is instructive to consider

$$T_4-T_3=\frac{\mu_A(x_i)-2\sqrt{\mu_A(x_i)\mu_B(x_i)}+\mu_B(x_i)}{6}=\frac{1}{6}(\sqrt{\mu_A(x_i)}-\sqrt{\mu_A(x_i)})^2\ge 0$$

for each $\mu_A(x_i), \mu_B(x_i)\epsilon[0, 1]$ with equality whenever $\mu_A(x_i) = \mu_B(x_i)$. The resulting inequality can be rescheduled as

$$\begin{aligned}
&T_4+1\ge T_3+1\Rightarrow \frac{T_0}{2}+1\ge\frac{T_0+T_1}{3}+1\\
&\Rightarrow\frac{2+T_0}{3+T_0+T_1}\ge\frac{2}{3}\Rightarrow\log_{\frac{3}{2}}\frac{2+T_0}{3+T_0+T_1}\ge\log_{\frac{3}{2}}\frac{2}{3}\\
&\Rightarrow(2+T_0)\log_{\frac{3}{2}}\frac{2+T_0}{3+T_0+T_1}\ge(2+T_0)\log_{\frac{3}{2}}\frac{2}{3} \qquad (2)
\end{aligned}$$

Replacment of $\mu_A(x_i), \mu_B(x_i)$ by their counterparts $(1 - \mu_A(x_i)), (1 - \mu_B(x_i))$ in the resulting inequality (4) yields

$$(4-T_0)\log_{\frac{3}{2}}\frac{4-T_0}{5-T_0+T_2}\ge(4-T_0)\log_{\frac{3}{2}}\frac{2}{3} \qquad (3)$$

We can add the aforementioned inequalities (4) and (5) to get the proclaimed result. Furthermore,

Theorem 3.3 Show that : $0 \leq K_{FS}(A,B) \leq 6\log_{\frac{3}{2}}\left(\frac{9}{8}\right)n$

Proof.It is interesting to note that $K_{FS}(A,B)$ remains unchanged B is replaced with A^c. Thus,

$$\begin{aligned}
& K_{FS}(A,A^c) \\
= & \sum_{i=1}^{n}\left[3\log_{\frac{3}{2}}\left(\frac{3}{\frac{2}{3}\left[4+\sqrt{\mu_A(x_i)-\mu_A^2(x_i)}\right]}\right)+3\log_{\frac{3}{2}}\left(\frac{3}{\frac{2}{3}\left[4+\sqrt{\mu_A(x_i)-\mu_A^2(x_i)}\right]}\right)\right] \\
= & \sum_{i=1}^{n}6\log_{\frac{3}{2}}\left(\frac{3}{\frac{2}{3}\left[4+\sqrt{\mu_A(x_i)-\mu_A^2(x_i)}\right]}\right)=\sum_{i=1}^{n}6\log_{\frac{3}{2}}\left(\frac{\frac{9}{8}}{1+\frac{1}{4}\sqrt{\mu_A(x_i)-\mu_A^2(x_i)}}\right) \\
= & \sum_{i=1}^{n}\left[6\log_{\frac{3}{2}}\frac{9}{8}-6\log_{\frac{3}{2}}\left[1+\frac{1}{4}\sqrt{\mu_A(x_i)(1-\mu_A(x_i))}\right]\right] \\
\Rightarrow & \; K_{FS}(A,A^c)=6n\log_{\frac{3}{2}}\frac{9}{8}-6H_F(A) \qquad (4)
\end{aligned}$$

With the aid of non-negative conditions;$H_F(A) \geq 0 \forall \mu_A(x_i)\epsilon[0,1]$ and $K_{FS}(A,B) \geq 0 \forall \mu_A(x_i),\mu_B(x_i)\epsilon[0,1]$, the resulting equality (6) yields

$$\begin{aligned}
& H_F(A)=\left(2\log_2 2-\log_{\frac{3}{2}}2\right)n-\frac{1}{6}K_{FS}(A,A^c)\geq 0 \\
& \Rightarrow 0 \leq K_{FS}(A,A^c) \leq 6\log_{(\frac{3}{2}}\left(\frac{9}{8}\right)n \qquad (5)
\end{aligned}$$

Discussion. With the aid of inequality (7), it is submitted that $K_{FS}(A,A^c)$ is finite quantity for each fixed n. Equivalently, readers can verify that $K_{FS}(A,B)$ also satisfies the same range value i.e. $0 \leq K_{FS}(A,B) \leq 6\log_{(\frac{3}{2}}\left(\frac{9}{8}\right)n$. Also, the three- dimensional region plot $K_{FS}(A,B)$, shown in Fig.3, justifies our claims

We shall now provide the applicability of newly discovered symmetric fuzzy cross entropy measure by solving some MADM problem as follows.

4. Application to Multiple Attribute Decision Making problems:

The existing methods for solving MADM problems mainly rely on the decision maker's decision as he is free to provide his judgement based on the suitable alternatives[3,4,6,7,9,10,13,17].In solving MADM problems, the DM has to reckon the best alternative, though it may not not exist in real situations. With

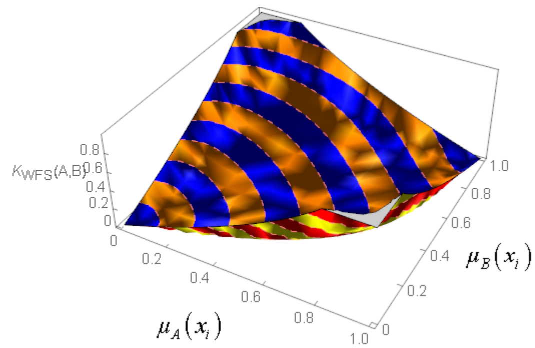

Figure 2: Concavity Property Exhibited by $H_F(A, B)$

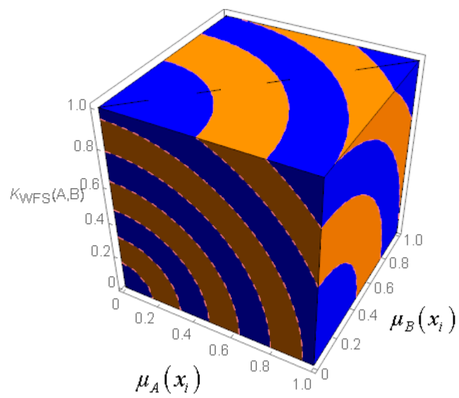

Figure 3: Concavity Property Exhibited by $H_F(A, B)$

this idea in mind under fuzzy environment , the selection of relative fuzzy ideal alternative vector, denoted by A^+, is traditionally based on following fuzzy ideal value μ_j^+.

$$\text{Let} \quad A^+ = (< x_1, \mu_1^+ >, < x_2, \mu_2^+ >, < x_3, \mu_2^+ >, ..., < x_{n-1}, \mu_{n-1}^+ >, < x_n, \mu_n^+ >) \tag{6}$$

$$\text{where } \mu_j^+ = max_i\{\mu_{ij}\}; i = 1, 2, 3, ..., m \text{ and} j = 1, 2, 3, ..., n. \tag{7}$$

Here, μ_j^+ is termed as fuzzy ideal value.

But due to ambiguity in human thinking and time pressure, the traditional approach of selecting a relative fuzzy ideal alternative vector is not suitable as it reflects ambiguity in the subjective judgement made by the rational DM.To get rid off this conflicting situation, this study established some novel relative fuzzy weighted ideal alternative vectors(RFWIA) based on fuzzy Archimedean, Einsteen and Hammer weighted averaging aggregation operators[] as under. I. The RFWIA vector based upon fuzzy Archimedean weighted averaging aggregation (FAWA) operator can be denoted by $A_W^{A^+}$ and is given as

$$A_W^{A^+} = (< x_1, \mu_1^{A+} >, < x_2, \mu_2^{A+} >, ..., < x_n, \mu_n^{A+} >) \text{ where} \quad (8)$$

$$\mu_j^{A+} = 1 - \Pi_{i=1}^m (1-\mu_{ij})^{w_j}, \text{ for a given } j = 1, 2, ..., n. \quad (9)$$

II. The RFWIA vector based upon fuzzy Einsteen weighted averaging aggregation (FEWA) operator can be denoted by $A_W^{E^+}$ and is given as

$$A_W^{E^+} = (< x_1, \mu_1^{E+} >, < x_2, \mu_2^{E+} >, ..., < x_n, \mu_n^{E+} >) \text{ where} \quad (10)$$

$$\mu_j^{E+} = \frac{\Pi_{i=1}^m (1+\mu_{ij})^{w_j} - \Pi_{i=1}^m (1-\mu_{ij})^{w_j}}{\Pi_{i=1}^m (1+\mu_{ij})^{w_j} + \Pi_{i=1}^m (1-\mu_{ij})^{w_j}} \text{ for a given } j = 1, 2, ..., n. \quad (11)$$

III. The RFWIA vector based upon fuzzy Hammer weighted averaging aggregation (FHWA) operator can be denoted by $A_W^{H^+}$ and is given as

$$A_W^{H^+} = (< x_1, \mu_1^{H+} >, < x_2, \mu_2^{H+} >, ..., < x_n, \mu_n^{H+} >) \text{ where} \quad (12)$$

$$\mu_j^{H+} = \frac{\Pi_{i=1}^m (1+(\gamma-1)\mu_{ij})^{w_j} - \Pi_{i=1}^m (1-\mu_{ij})^{w_j}}{\Pi_{i=1}^m (1+(\gamma-1)\mu_{ij})^{w_j} + (\gamma-1)\Pi_{i=1}^m (1-\mu_{ij})^{w_j}}, \quad (13)$$

for a given $j = 1, 2, .., n, \gamma \epsilon (0, \infty), \gamma \neq 1$

5. A Fuzzy Cross Entropy Based Method for Solving MADM problem:

Suppose there are m known potential alternatives, denoted by the set $A = (A_1, A_2, ..., A_m)$ where $(m \geq 2)$. Also suppose that there are $n (n \geq 2)$ known attributes denoted by the set $G = (G_1, G_2, ..., G_n)$.. Let the weights corresponding to each attribute are denoted by $w = (w_1, w_2, ..., w_n)^T$ such that each w_j is non-negative for $1 \leq j \leq n$ and satisfy $\sum_{i=1}^n w_j = 1$. Here, the entity w_j represents weight of the $j^t h$ attribute G_j.

We can assume $A = \{t_{ij}\}_{m \times n}$ to be the decision matrix as given by the rational decision maker. Our aim is to reckon the best alternative while solving

ISSN (ONLINE) 2583-2506
http://publications.rayatbahrauniversity.edu.in
RBIJMR-Rayat Bahra International Journal of Multidisciplinary Research, Vol. 01, Issue 01, December 2021

a given MADM problem under the available fuzzy preference information on each alternative.

Step-I:- Since the attributes, in general, are non-commensurate and conflicting, it becomes exigency for us to get the normalized form of the given decision matrix. This will transform the various attributes values into fuzzy numbers.

Let $D = \{\mu_{ij}\}_{m\times n}$ be the corresponding matrix obtained after normalization. The matrix D here is termed as fuzzy decision matrix where each μ_{ij} represents true membership degree that the alternative A_i satisfies the attribute G_j where $0 \leq \mu_{ij} \leq 1$ and each μ_{ij} is obtained deploying the linear transformation[10]

$$\mu_{ij} = \begin{cases} \frac{t_{ij}-{t_j}^{\min}}{{t_j}^{\max}-{t_j}^{\min}} & \text{for benefit attribute,} \\ \frac{{t_j}^{\max}-t_{ij}}{{t_j}^{\max}-{t_j}^{\min}} & \text{for cost attribute,such that} \end{cases}$$

${t_j}^{\max} = max\{t_{1j}, t_{2j}, ..., t_{mj}\}; {t_j}^{\min} = min\{t_{1j}, t_{2j}, ..., t_{mj}\}; j = 1, 2, ..., n.$

Step-II:- For the given alternatives $A_i(i = 1, 2, ..., m)$, define relative (weighted) fuzzy ideal alternative vectors A^+ ,$A_W^{A^+}$,$A_W^{E^+}$, and $A_W^{H^+}$ by employing equations (10-15) .

Step-III:- Calculate the cross entropy measure $K_{WFS}(A_i, A^+)$ values between each alternative A_i and A^+ by using the (17), which can be obtained by replacing $\mu_A(x_i), \mu_B(x_i)$ with μ_{ij}, μ_j^+ respectively. Thus,

$$K_{WFS}(A_i, A^+) = \sum_{j=1}^{n} w_j \left[(2 + \mu_{ij} + \mu_j^+) \log_{\frac{3}{2}} \frac{2 + \mu_{ij} + \mu_j^+}{\frac{2}{3}[3 + \mu_{ij} + \mu_j^+ + \sqrt{\mu_{ij}\mu_j^+}]} \right.$$
$$\left. +(4 - \mu_{ij} - \mu_j^+) \log_{\frac{3}{2}} \frac{4 - \mu_{ij} - \mu_j^+}{\frac{2}{3}[5 - \mu_{ij} - \mu_j^+ + \sqrt{(1 - \mu_{ij})(1 - \mu_j^+)}]} \right] \quad (14)$$

Step-IV:- Based upon the minimum cross entropy value, each alternative $A_i(i = 1, 2, ..., m)$ can be classified from best to worse in accordance with Minimum Argument Principle. Thus, Min.$K_{WFS}(A_i, A^+)$ value indicates that the alternative A_i is close to relative fuzzy ideal alternative vector A^+. Similarly,calculate the symmetric fuzzy cross entropy measure $K_{WFS}(A_i, A_w^{A^+})$,$K_{WFS}(A_i, A_w^{E^+})$ and

ISSN (ONLINE) 2583-2506
http://publications.rayatbahrauniversity.edu.in
RBIJMR-Rayat Bahra International Journal of Multidisciplinary Research, Vol. 01, Issue 01, December 2021

$K_{WFS}(A_i, A_w^{H^+})$ values which can be computed by replacing μ_j^+ either by μ_j^{A+}, or μ_j^{E+} or μ_j^{H+} in(17) and classify the corresponding alternative A_i accordingly.

To illustrate the applicability of the proposed fuzzy cross entropy based methodology, we shall solve a classical MADM problem based financial strategy as follows.

Example A manufacturer has planned its financial strategy for the forthcoming assessment year according to firm's objective. After the preliminary screening, all the possible the attributes $G_j(j = 1, 2, 3, 4, 5)$ are obtained as follows.

G_1 : Output value is \$10000, G_2 : Investment cost is \$10000, G_3 : Total sales is \$10000, G_4 : Profit proportion of the company and G_5 : degree of environment pollution. It is assumed that the attributes with benefits are G_1, G_3 and G_4 whereas attributes with cost are G_2 and G_5 respectively.We can consider the weight vector as $w = (0, 0, 0.6618, 0.2344, 0.1038)^T$. Suppose there are four potential alternatives $A_i(i = 1, 2, 3, 4)$ along with five known attributes $G_j(j = 1, 2, 3, 4, 5)$ and the decision maker prepares the the following decision matrix $A = [a_{ij}]_{4\times5}$.

$$A = \begin{array}{c} \\ \end{array} \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 & G_5 \\ A_1 : & 8350 & 5300 & 6135 & 0.82 & 0.17 \\ A_2 : & 7455 & 4952 & 6527 & 0.65 & 0.13 \\ A_3 : & 11000 & 8001 & 9008 & 0.59 & 0.15 \\ A_4 : & 9624 & 5822 & 8892 & 0.74 & 0.12 \end{pmatrix}.$$

Step-I Normalize the given decision matrix A into fuzzy decision matrix D by using the linear transformation [10]. Thus,

$$D = \{\mu_{ij}\}_{4\times5} = \begin{pmatrix} 0.2525 & 0.8859 & 0 & 1 & 0 \\ 0 & 1 & 0.1364 & 0.2609 & 0.8 \\ 1 & 0 & 1 & 0 & 0.4 \\ 0.6118 & 0.7147 & 0.9596 & 0.6521 & 1 \end{pmatrix}.$$

Step-II All the four possible alternatives A_1, A_2, A_3, A_4 can be obtained from the fuzzy decision matrix D of Step 1. The desired relative fuzzy ideal alternative vector A^+ can be calculated employing (8-9):

$$
\begin{aligned}
A_1 &= (< x_1, 0.2525 >, < x_2, 0.8859 >, < x_3, 0.0000 >, < x_4, 1.0000 >, < x_5, 0.0000 >)\\
A_2 &= (< x_1, 0.0000 >, < x_2, 1.0000 >, < x_3, 0.1364 >, < x_4, 0.2609 >, < x_5, 0.8000 >)\\
A_3 &= (< x_1, 1.0000 >, < x_2, 0.0000 >, < x_3, 1.0000 >, < x_4, 0.0000 >, < x_5, 0.4000 >)\\
A_4 &= (< x_1, 0.6118 >, < x_2, 0.7147 >, < x_3, 0.9596 >, < x_4, 0.6521 >, < x_5, 1.0000 >)\\
A^+ &= (< x_1, 1.0000 >, < x_2, 1.0000 >, < x_3, 1.0000 >, < x_4, 1.0000 >, < x_5, 1.0000 >)
\end{aligned}
$$

Step-III The symmetric fuzzy cross entropy measure $K_{WFS}(A_i, A^+)(i = 1, 2, 3, 4)$ between each alternative and relative fuzzy ideal alternative vector A^+ can be evaluated employing (16) and are as under.

$K_{WFS}(A_1, A^+) = 1.33439$, $K_{WFS}(A_2, A^+) = 0.93051$,

$K_{WFS}(A_3, A^+) = 0.47342$, $K_{WFS}(A_4, A^+) = 0.098216$

Step-IV. We can classify all the four alternatives in accordance with the minimum cross entropy measure $K_{WFS}(A_i, A^+)$. The best alternative is A_4 owing to the smallest value 0.098216. Hence, the optimum ranking order in this case is $A_4 \succ A_3 \succ A_2 \succ A_1$. (See Table.2) Also, the ranking order, best and worse alternative for the MADM problem under discussion obtained by the existing discrimination information measures ${}^jD_{FS}(A, B)(j = 1, 2)$ as well as by the proposed symmetric fuzzy cross entropy measures $K_{WFS}(A_i, A_w^{A^+})$, $K_{WFS}(A_i, A_w^{E^+})$ and $K_{WFS}(A_i, A_w^{H^+})$ are summarized in Tables.2 and Table.3 respectively.

Analysis and Discussion:- A careful analysis of the results depicted in Table.2 reveal that if the relative fuzzy ideal alternative vector is A^+, then,the existing fuzzy cross entropy measures ${}^jD_{FS}(A, B)(j = 1, 2)$ are not capable of classifying the alternatives. The ranking order obtained by the existing method [23] is $A_4 \succ A_3 \succ A_1 = A_2$ (Table. I) . These results create some ambiguity in the ranking order and hence it becomes exigency to identify and fix the actual cause of this information. However, the ranking order obtained by the proposed cross entropy measure $K_{WFS}(A_i, A^+)$, $K_{WFS}(A_i, A_w^{A+})$, $K_{WFS}(A_i, A_w^{E+})$ and $K_{WFS}(A_i, A_w^{H+})$ is $A_4 \succ A_3 \succ A_2 \succ A_1$ whereas the best and worse alternative

Table 1: Ranking order, best and worse lternative obtained by the proposed symmetric fuzzy cross entropy measure based upon WFA operators.

Cross entropy	Cross entropy Values				Ranking order	Best	Worse
	A_1	A_2	A_3	A_4			
$^1D_{FS}(A_i, A^+)$[2]	–	–	–	–	–	–	–
$^2D_{FS}(A_i, A^+)$[16]	–	–	–	–	–	–	–
$K_{WFS}(A_i, A^+)$	1.3344	0.9305	0.4734	0.0982	$A_4 \succ A_3 \succ A_2 \succ A_1$	A_4	A_1
$K_{WFS}(A_i, A_w^{E+})$	0.5905	0.8435	0.5105	1.1741	$A_3 \succ A_1 \succ A_2 \succ A_4$	A_3	A_4
$K_{WFS}(A_i, A_w^{H+})$	0.4322	0.3508	0.3364	0.5521	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3	A_4

are A_4 and A_1 respectively. This justifies the effectiveness and capability of the proposed symmetric fuzzy cross entropy measure. Hence, the final ranking should be $A_4 \succ A_3 \succ A_2 \succ A_1$ but cannot $A_4 \succ A_3 \succ A_1 = A_2$ as claimed by [23].It is concluded that our fuzzy cross entropy based MADM problem solving methodology is efficacious for handling non-commensurate and conflicting data in comparison with the existing method [23].

6. Conclusion:-

The proposed method for solving MADM problems with the fuzzy preference information on alternatives differ from the existing approach which assumes the relative fuzzy ideal alternative vector based on traditionally selecting the fuzzy ideal value for selecting the best alternative. The new symmetric weighted fuzzy cross entropy values between the known potential alternatives and RFWIA vectors based on various FWA operators reveal that the linear goal programming model [23] hide some useful evaluation information of the optimal alternative and thus affects the ranking analysis resulting in catastrophic economical disorder.The newly constructed RFWIA vectors based on fuzzy Archimedean, Einsteen and Hammer weighted averaging aggregation operators give more information. Hence, the proposed method fairly furnishes the consistent and feasible results and thus offers remarkable and effective evaluation information of MADM problems with the fuzzy preference information on alternatives. .Furhter, the proposed method can be utilized to obtain the optimal solution for MADM problems , which arise in other areas such as bearing fault analysis, image segmentation, pattern recognition, medical diagnosis, fault diagnosis of turbine etc. In the future, we will continue working to develop more symmetrical discrimination information measures and find their applications to improve the accuracy.

References

[1] Bernardo,J.J.,& Blin.J.M., *"A programming model of consumer choice among multi-attributed brands"*, Journal of Consumer Research,4(**1979**),p.111-118.

[2] Bhandarin.D. and Pal.N.R., *"Some new information measures for fuzzy sets "* , Information Sciences,67(**1993**),p.209-226.

[3] Calpine.H.C.,& Golding.A., *"Some properties of pareto-optimal choices in decision problems"* , OMEGA,4(**1976**),p.141-147.

[4] Chen.S.J.,& Hwang.C.L., "Fuzzy multiple attribute decision making methods and applications", New York:Springer(**1992**).

[5] Chiclanna.F.,Herrerea.F.,&Herrerea.-Viedma.E.., *"Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relation"*, Fuzzy Sets and Systems,7(**1998**),p.33-48.

[6] Dasarathy.B.V., *"Similarity measure anchored ranking technique for the analysis for the multidimensional data analysis"*, IEEE Transaction on Systems.Man. and Cybernetics ,6(**1976**),p.708-711.

[7] Eilon.S., "Goals and constraints in decision making ", Operational Research Quarterely,23(**1972**),p.3-15.

[8] Fan.Z.P.,Jian.Y.P.,& Xiao.S.H., *"Consistency of fuzzy judgement matrix and its properties"*, Control and Decision,16(1)(**1976**),p.69-71.

[9] Foerster.J.F., *"ModeMode coice decision process models: A comparision of compensatory and non-compensatory structures"*, Transportation Research,13(A)(**1979**),p.17-28.

[10] Hwang.C.L.,& Yoon.K., " Multiple attribute decision making: Methods and applications", Berlin:Springer(**1981**)

[11] Elion.S., "Goal programming and extensions " , MA:Lexington D.C.Health(**1976**).

[12] Kacprzyk.J., *"Group decision making based with a fuzzy linguistic majority preference relation"*, Fuzzy Sets and Systems,18(**1998**),p.105-118.

[13] Kruskal.J.B., *"Nonmetric multidimensional scaling:A numerical method "*, Psychometrika,29(**1998**),p.115-129.

[14] Ma.J.Fan.,Z.P.& Huang.L.H., *"A subjective and objective integrated approach to determine attribute weights"*, European Journal of Operational Research,112(**1999**),p.397-404.

[15] Orlovski.S.A., *"Decision making with a fuzzy preference relation"*, Fuzzy Sets and Systems,1(**1978**),p.155-167.

[16] Shang.X.G.,Jian.W.S., *"A note on fuzzy information measures"*, Pattern Recognition Letters,18(**1997**),p.425-432.

[17] Srinivasan.V.& Shocker.A.D., *"Linear programming techniques for multidimensional analysis of preference"*, Psychometrika,38(**1973**),p.337-369.

[18] Tanino.T., "On group decision making under fuzzy preference", Multiperson decision making using fuzzy sets and possibility theory,Kluwer Academic Publishers,(**1990**),p.172-185.

[19] Wan.S.P.,Xu.G.L.,Wang.F.,Dong.J.Y., *"A new method for Attanassov's interval-valued intuitionistic fuzzy numbers with incomplete attribute weight information"*Information Sciences,316(**2015**),p.329-347.

[20] Wei.G.W.,*"Picture fuzzy cross entropy for multiple attribute decision making problems"*Rus.Econ.Manag,17(64)(**2016**),p.491-502.

[21] Xu.Z.S.,&Yager.R.R., *"Some geometric aggregation operators based on intuitionistic fuzzy sets"*, Int.J.Gen.Syst35,(**2006**),p.417-433.

[22] Zadeh.L.A., *"Fuzzy sets"*,Information and control,8(**1965**),p.338-353.

[23] Zhi-Ping Fan,Guo-Fen Hu.,Si-Han Xiao., *"A method for multiple attribute decision making with the fuzzy preference relation on alternatives"* , Computers and Industrial Engineering 46,(**2003**),p.321-327.