ISSN (ONLINE) 2583-2506
http://publications.rayatbahrauniversity.edu.in
RBIJMR-Rayat Bahra International Journal of Multidisciplinary Research, Vol. 02, Issue 02, December 2022

Discontinuity in Nuclear Charge Radii of Different Isotopes

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Abstract: *In this work we compare two relativistic mean field (RMF) approaches to analyze the charge radii of the different isotope of different nuclei. The systematic trend in charge radii of different isotope is of great interest due to its distinctive aspect at the nucleon-shell closure and the odd-even staggering (OES) behaviour. The modified RMF model is more promising to estimate the root mean square (rms) charge radii and the calculations are extended to the study of heavier odd-Z Copper (Cu) and Indium (In) isotopic chain. The neutron-proton short range correlation (np-SRCs) is analyzed, which shows that the np-SRCs plays an indispensible role to determine the fine structure of nuclear charge radii along the isotopic chain quantitatively.*

Keywords: *Nuclear Structure, Single Particle Energy, Relativistic Mean Field, Staggering, Charge radii, root mean square radii*

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ISSN (ONLINE) 2583-2506
http://publications.rayatbahrauniversity.edu.in
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1. Introduction

Hans Geiger and Ernest Marsden predicted the nuclear charge radius in 1909. Charge radii are the fundamental quantity to describe the atomic nuclei. The empirical relation between the charge radius (R) and the mass number (A) for heavier nuclei (A > 20) is $R \approx r_0 A^{1/3}$ [1, 2].

Nuclear size and density distribution are important bulk properties of nuclei that determine the nuclear potential, single-particle orbitals, and wave function. Also, nuclear charge radius is very useful to find out the properties of nucleus. Nucleus charge radius of different nuclei and their isotopes shows some discontinuity in charge radii, the variation is known as odd-even staggering [3, 4]. Charge radii of many atomic nuclei such as Calcium, Cadmium, Tin, Mercury, Copper, and Potassium isotopes have been proven to be far away from the beta-stability line, measured with high precision [5, 6]. However, there are some specific nuclei, in which large discrepancies in experimental measurements and theoretical predictions have observed. For example, a parabolic-like shape and strong odd-even staggering (OES) effects have long been known to exist in the calcium isotopes between ^{40}Ca and ^{48}Ca. Such feature persists toward the neutron-deficient region [7]. Beyond N = 29, the charge radii increase rapidly and the radius of ^{52}Ca is remarkably large as compared to ^{48}Ca. This is unexpected because N = 32 has been believed to be a magic number in the calcium isotopes [8, 9].

Comparing the charge radii of calcium isotopes with potassium isotopes, the amplitude of the parabolic-like shape between ^{39}K and ^{47}K is smaller due to the last unpaired proton [10]. Meanwhile, rapid increase of the charge radii is also found across the N = 28 shell closure [11]. The neutron-rich shell closure at N=32 found in the potassium isotopic chain was investigated in Refs. [9, 12], which shows relatively enhanced stability. Recently, the collinear resonance ionization spectroscopy technique is employed to measure the charge radii of potassium isotopes [5], and the precision measurement of charge radii beyond N = 32 is firstly performed below Z < 20 for potassium isotopes [6]. No sudden increase in the charge radius of ^{52}K is observed. All these results challenged our understanding of the evolution of nuclear charge radii of exotic isotopes with large neutron or proton excess. To address these challenges, numerous approaches have been proposed so far. In Ref. [13], a statistical method is introduced to study nuclear charge radii by combining sophisticated nuclear models with the naive Bayesian probability classifier. This method forecasts a rapid increase of charge radii beyond N = 28. In Ref. [14], a feed-forward neural network model which relates the charge radii to the symmetrical energy is explored. The vigorous increase in the charge radii beyond N = 28 is well reproduced by the Fayans energy density functional (EDF) model [6, 15]. However, this method amplifies the OES effect of the charge radii of the Potassium isotopic chain. In addition, the deviation between experiments and theory becomes larger toward the neutron-deficient region [6]. In Ref. [16], an empirical ansatz formed on the relativistic mean field (RMF) theory was proposed, which adds a correction term induced by the difference of pairing interactions for protons and neutrons calculated self-consistently in the RMF.

This modified approach can be used to remarkably reproduce the OES effects of charge radii of the calcium isotopes along with nine other even-Z isotopic chains, particularly the substantial increase of charge radii around N = 28 shell closure across the calcium isotopic chain. Here, we would like to extend the ansatz of Ref. [16] to analyse the root mean square (rms) charge radius of the odd-proton potassium isotopes. The staggering reflect the fact that the nuclear charge radii of odd neutron isotopes are smaller than the average of their even- neutron neighbours.

Firstly charged radius is calculated by $R \approx r_0 A^{1/3}$ [1, 2] but in this there is plenty of discrepencies between experimental and theoretical results found in proton or neutron rich region. Despite this, the energy density functional theories, the non-relativistic Hartree-Fock-Bogoliubov model [17, 18] and the relativistic mean-field (RMF) theory [19, 20] provide the nuclear charge radii self consistently. But both model failed quantitatively to describe the fine structure of isotopic chain. Plenty of experiments are employed to find out the fine structure of nuclear charge radii. But most of the experiments gives the influence on nuclear size, quadrapole vibrations [21]. The EDF model modified the charge radius formula under the RMF model. This modified RMF model will be able to describe the behaviour of charge radii is essential to understand the nuclear force. This correction term induced by the difference of pairing interaction between proton and neutron calculated self consistently.

2. Odd-even staggering behaviour in nuclear size

The OES behaviour of nuclear mass are generally observed throughout the nuclear chart. The same scenario can be encountered in nuclear charge radii [1-3]. This staggering reflects the facts that the nuclear charge radii of odd-neutron isotopes are smaller than the averages of their even-neutron neighbours. There are many possible mechanisms have been proposed to interpret the OES effects of nuclear charge radii, such as quadrupole vibrations by the odd neutron, core polarization by valence neutrons, four-particle correlations or α-particle clustering, three- or four-body effective residual interaction, the special deformation effects and neutron pairing energy, etc. The location of last occupied nucleon plays an important role for describing the charge radii, for example the unpaired neutron can affect the deformation and the sign of OES of charge radii.

Nevertheless, the charge radii of copper and indium isotopes involving unpaired proton are also carefully calculated by blocking approximation.

3. Shell structure in charge radii

Here, we focus on the systematic behaviour of nuclear charge radii along copper and indium isotopic chains. The pairing strength is determined through the mass staggering [24]. In order to reflect the universality of our results, the pairing strength V_0 = 350 MeV fm^3 is fixed with reference [25]. It is instructive to test whether this new ansatz can be extended to heavier odd-Z isotopic chains. The systematic trend in root-mean-square (rms) charge radii as a function of proton or neutron number exhibits a discontinuity at the nucleon-shell closures [2] the rms charge radii of copper and indium isotopes are obtained by the RMF(BCS) method and the modified RMF(BCS)* approach. It is very clear that without the correction term, RMF(BCS) method cannot describe the charge radii of copper isotopes, especially the hump behavior (or named as parabolic-like shape) between N = 28 and N = 50 region. With considering the correction term, the RMF(BCS)* reproduces the trend of experimental data as well. However the odd-even oscillations behaviour is slightly overestimated in $^{65-73}$Cu isotopes. This may originate from the last unpaired proton and neutron which violates the time-reversal symmetry [2]. For the lighter odd-Z potassium isotopes, the OES effect is still overestimated. As a comparison, additionally given are the outcomes of the HF (BCS) technique using the SVbas Skyrme parameterization. A bigger departure occurs in the neutron (proton)-rich region, especially at N = 28 and N = 50, but the HF(BCS) model appears to represent the minute fluctuations between N = 31 and N = 44 region.

The experimental data diverge between A = 104 and A = 122. However, the parabolic-like shape can also be reproduced by RMF(BCS)* approach. As encountered in Cu isotopes, HF(BCS) still overestimates the rms charge radii across strong neutron-closure shells. As well known, this parabolic-like shape of nuclear charge radii between two strong closure shells are observed generally along isotopic chains, such as in Ca, Cd, Sn, etc. in reference [26], it points out that the parabolic behaviour of the formation of the charged radii is caused due to a linear correlation between the charge radius and the corresponding quadrupole deformation.

It is clear, the np-SRC associated to Cooper pairs condensation plays dominant contribution. These nuclei with N = 50 and N = 82 closed-shells are more difficult to be excited than their neighbours, which is evidenced by their relatively high excitation energies and low excitation probabilities. The other intriguing phenomenon is about the rapidly increase of charge radii across the magic number. The abrupt change of charge radii is shown beyond N = 50 neutron-shell closure. The similar case is also studied earlier in RMF within NL-SH parameterization set [30]. This rapidly increasing still can be predicted by RMF(BCS)* approach across N = 82 magic number. These phenomena can be found dramatically in latest study of Cadmium [12], Tin [13], Mercury [14], isotopic chains, etc. The shell closures have been identified by this discontinuity in the slope of the rms charge radii as a function of the number of protons or neutrons in the nucleus. the pronounced kink at the N = 82 shell closure seen in various different isotopic chains is due to reduced neutron pairing. This has been attributed to the rather small isospin dependence of spin-orbit term in the RMF model. The emergency of simple and regular patterns of rapidly increasing of charge radii across N = 50 and N = 82 are common features observed in self-bound many-nucleon systems. The developed analytic response relativistic coupled-cluster theory has been applied to determine mass shift and field shift factors for different atomic states of indium [16]. In fact, the electromagnetic moments played a key role in motivating the most nuclear structure models. This simple picture of nuclear structure seems to be supported by a rather constant value of their nuclear moments, which present very small variations when neutrons are added. In this work, the entire indium isotopic chain almost show spherical shape. Precise knowledge of these radial moments is essential to establish reliable constraints on the existence of new forces from precision isotope shift measurements [17]. The new ansatz can describe the charge radii of Copper and Indium isotopic chains better than the one without modification. Especially, the parabolic-like shape are naturally shown between two strong neutron magic shells. This means the discontinuity feature is also seen across N = 50 and N = 82 closed-shells along copper and indium isotopic chains respectively. The results obtained by HF+BCS model failed to reproduce the discontinuity aspect for both of them. Odd-even staggering behavior in nuclear size as well known, the OES behavior of nuclear mass are generally observed throughout the nuclear chart. The same scenario can be encountered in nuclear charge radii [1]. This staggering reflects the facts that the nuclear charge radii of odd-neutron isotopes are smaller than the averages of their even-neutron neighbours. There are many possible mechanisms have been proposed to interpret the OES effects of nuclear charge radii, such as quadrupole vibrations by the odd neutron [21] core polarization by valence neutrons four-particle correlations or α-particle clustering [32], three- or four-body effective residual interaction, the special deformation effects [33, 34] and neutron pairing energy, etc. [35]. The location of last occupied nucleon plays an important role for describing the charge radii, for example the unpaired neutron can affect the deformation and the sign of OES of charge radii. The charge radii of copper and indium isotopes involving unpaired proton are also carefully calculated by blocking approximation.

4. Blocking Effect on Charge Radii

For example, the potential energy surfaces of ^{48}K, ^{50}K and ^{52}K, as function of the quadrupole deformation parameter, the plotting are β_{20}, with different assignments of the single particle orbits which are occupied by the last unpaired proton (Z) and the neutron (N). The occupied orbits, given by the combinations of spherical (s, p, d, f) and Nilsson quantum numbers [N, n_z, m_l] in the square brackets. Where N is main quantum number and n_z is the projection of N on the z-axis, m_l is the component of the orbital angular momentum [36]. Below, for convenience of the discussion, using an expression such as ($1d_{3/2}$, $2p_{3/2}$) to denote the orbitals occupied by the last unpaired proton (first term in the bracket) and the neutron (last term in the bracket). As a general idea, the occupation of the single particle level is determined to be self-consistent resulting to obtain the largest binding energy. Similarly, the last unpaired proton is found to occupy the $1d_{3/2}$ orbital. The last unpaired neutron in ^{48}K and ^{50}K is found to occupy the $2p_{3/2}$ orbital, while that in ^{52}K, it is found to occupy the $1f_{5/2}$ orbital. These configurations yield the largest binding energy. Moreover, the potential energy surfaces are relatively soft. This implies that beyond mean field studies, which take into account configuration mixing, might be needed to describe correctly these nuclei. For ^{48}K, the configuration ($1d_{3/2}$,$1f_{5/2}$) is found to show a spherical ground state, but the corresponding charge radius is found to be much smaller than the experimental radii. If the configuration ($1d_{3/2}$, $1f_{7/2}$) is chosen, the binding energy of ^{48}K is smaller by 2 MeV and the charge radius is increased to 3.487 *fm*. Additionally, the configuration of ($1f_{7/2}$, $2p_{3/2}$) advances to a smaller binding energy than the ($1d_{3/2}$, $2p_{3/2}$) configuration comparatively, and the charge radius of 3.481 *fm*, agreeing with result of the experiment. Therefore, for N = 29, we can make a conclusion that either the last unpaired nucleons prefer to occupy the ($1f_{7/2}$, $2p_{3/2}$) states instead of the self-consistent ($1d_{3/2}$, $2p_{3/2}$) configuration or the nucleus is deformed with a $\beta_{20} \approx -0.2$. We cannot realize both of the scenarios in the present self-consistent calculation. Similar conclusions can be drawn for ^{50}K. For ^{52}K, from the energy point of view, the most favored configuration is ($1d_{3/2}$, $1f_{5/2}$). There are relatively small differences between all the four configurations. Both, ($1d_{3/2}$, $2p_{3/2}$) and ($1f_{7/2}$, $1f_{5/2}$) are known to yield results in reasonable agreement with data. The single particle levels occupied by the neutron and the last unpaired proton beyond N = 28, are assigned as explained above. For ^{48}K and ^{50}K, the ($1f_{7/2}$, $2p_{3/2}$) configurations are used. For ^{52}K, the configuration ($1f_{7/2}$, $1f_{5/2}$) is used. So, the theoretical results are in a better agreement with the data for both, rapid increase of charge radius and the OES effect. It should be noted that this do not simply states that the neutron and the last unpaired proton really occupy these orbitals. It can be seen as a more convenient way to choose the deformation of the studied nuclei. Next we check whether the further experimental information, such as spin and parity, can help to determine which single particle configurations are preferred. The HF model very closely describes the nuclear charge radii, but it shoes large deviation in neutron and proton rich region. So the HF model also fails to reproduce the discontinuity aspect. In the modified RMF approach, strength of the pairing interaction is decided by fitting in the odd-even staggering of the binding energies. Adversely, the following three-point formula is employed;

$$\Delta E = \frac{1}{2}[B(N-1,Z) - 2B(N,Z) + B(N+1,Z)] \quad (1)$$

(where the binding energy for a nucleus of the neutron number N and proton number Z is B (N, Z). In the current study, the pairing strength is fixed at 350 MeV fm^3 and the pairing space is set to include all of the single particle levels within 24 MeV below and above the Fermi surface. For studying the nuclei with odd number of nucleons, a blocking approximation is embraced. The last single particle level occupied by the odd nucleon is blocked during each iteration of the self-consistent process used to solve the RMF equations. It should be noted that such a process may not produce correct findings in some circumstances, such as when the single particle levels around the Fermi surface are dense or configuration mixing is extremely important. So, the modified RMF approach gives the result in isotopic chain same as the OES. So it is necessary to give a unified approach for describing the charge radii of finite nuclei over the entire nuclear chart.

5. Double odd-even staggering effects

Similar to binding energies, we can also determine a three-point formula to extract the OES for a charge radii;

$$\Delta r = \frac{1}{2}[R(N-1,Z) - 2R(N,Z) + R(N+1,Z)] \quad (2)$$

where R(N, Z) will be the root mean square charge radius. Noticeably, the OES effects of charge radii, i.e., the nuclear charge radii of odd-neutron isotopes are smaller than the average of their even-neutron neighbours, have been observed throughout the nuclear chart. Numerous possible explanations for this have already been given, such as, blocking of the ground state quadrupole vibrations by the odd neutron [21], core polarizations by valence neutron, α-cluster, three- or four-body residual interactions [37, 38], special deformation effects [34, 35], neutron pairing energies, pairing correlation, etc. Therefore, it is worth checking whether this empirical ansatz can provide a reasonable description of the OES of charge radii of odd (Z =19) potassium isotopes. The OES of the binding energies (upper panel) and the charge radii (lower panel) are compared with the experimental data. The RMF(BCS) /RMF(BCS)* method can reproduce the OES of binding energies efficiently.

As for the potassium isotopic chain, the general trend of the OES of charge radii calculated by Eq. (2) is forged as well.

However, the RMF(BCS)* method amplifies the OES of charge radii, especially in the neutron-rich region, as can be inferred already taken For ^{37,38}K, the OES behaviors are reversed due to the slightly overestimated charge radii of ^{36,38}K. In principle, the overestimation of OES can be easily corrected by reducing the coupling a0, which is determined by fitting to the charge radii of calcium isotopes [16]. We will not perform such a fine tuning in this work as we are not aiming at a perfect fit of the experimental data. As one can see from the charge radii of ^{50}K and ^{52}K are misjudged, specially, in both the RMF(BCS)* approach and the Fayans density functional theory. Similar deviation may be due to the blocking effect of unpaired nucleons, as mentioned above. Along with the isotopic chain, the added neutrons are mainly located in the outer edge of the nucleus for neutron-rich isotopes. As stressed in reference [22] neutron-proton pairing correlation can cause proton to advance towards the added neutrons and increase the nuclear charge radius. Specifically for the unpaired proton and neutron, the np-pairs could play a larger role. As a result, in the following, we will study the blocking treatment of the last unpaired proton/neutron.

6. Conclusion

In the present work we analysed the nuclear radii of various isotopes investigated in literature and the main focus in that work is on the systematic behaviour of nuclear charge radii of isotope of different nuclei such as Copper, Indium, Potassium, by odd even staggering. The rms charge radius of proton or neutron numbers has some discontinuities. By plotting the graph between neutron number and rms charge radius for both the RMF approach. There is some difference between the RMF approach and modified RMF in isotopes of such nuclei.

The unmodified RMF approach cannot describe the charge radii of copper, indium isotope. The charge radii of isotope of such nuclei involving unpaired proton are calculated by blocking approximation.

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