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Neutron Star Properties in Relativistic Mean-Field

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Abstract: *The formation of neutron stars, originating from the collapse of massive star cores, is a remarkably dense object with strong gravitational and magnetic fields. We solve the coupled relativistic mean-field equations and nucleon dynamics. Incorporating equations of hydrostatic equilibrium, continuity of mass, we model neutron star structure. We present pressure and mass profiles, alongside computations of key parameters like total mass and radius, within relativistic framework.*

Key words: *Neutron star, Gravitational Field, Relativistic Mean Field, Equation of State*

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1. Introduction

In addition to their independent, fundamental significance, studies of the structural characteristics and composition of the constituents of matter at extremely high densities and temperatures play an extremely important role in clarifying the physical nature of the internal structure and integral parameters of neutron stars. A quantum field approach in the framework of quantum hadrodynamics (QHD) provides a fairly adequate description of the properties of nuclear matter and of finite nuclei, treating them as a system of strongly interacting baryons and mesons. One theory of this type that has effective applications, is the relativistic mean-field theory [1-3]. This theory yields satisfactory descriptions of the structure of finite nuclei [4], the equation of state of nuclear matter [5], and the features of heavy ion scattering [6]. The parameters of the mean-field model characterizing the interaction of a nucleon with σ,ω, and ρ mesons can be self consistently determined starting with empirical data on symmetric nuclear matter near the saturation density. This, in turn, leads to the possibility of obtaining equations of state for superdense, isospin-asymmetric nuclear matter. In these studies it has been assumed that the masses of the scalar-isoscalar (σ), vector-isoscalar (ω), and vector-isovector (ρ) mesons and their coupling constants are independent of the density and of the values of the fields. In addition, the δ -scalar meson (a0 (980)) is not included among the exchange mesons. Relativistic mean-field theory models have been constructed [7, 8] assuming that the nucleon and exchange meson masses in nuclear media obey the Brown-Rho scaling law [9]. The results showed that including the density dependence of the mass leads to a more rigid equation of state for the matter.

2. Neutron Star History

Neutron stars are best described as remnants or corpses of stars that have reached their end. In 1934, the existence of neutron star was first hypothesized by Walter Baade and Fritz Zwicky. By the year 1939, the theoretical calculations were carried out by J. Robert Oppenheimer and George M. Volkoff, using general structure equations and formulated the new equations now known as the Tolman-Oppenheimer-Volkov (TOV) equations. These theoretical assumptions were confirmed when Jocelyn B. Burnell and Antony Hewish first observed a type of neutron star, called pulsars, in 1967 [10]. This was ground breaking discovery with multitudes of implications for research. As one of the densest bodies to exist in the universe, neutron stars are a curious existence and were prime subjects to test Albert Einstein's theory of Relativity. However, due to the limitations of research that can be done with these celestial bodies, most researchers utilize general equations of states and structure to create simulated models to compare to the existent neutron stars [11].

The initial mass of a star has a significant impact on its evolution. After the main-sequence evolution, a star becomes a white dwarf, a neutron star, or a black hole [12]. Stars which are tens of times more massive than the Sun can fuse elements up to iron if their mass is above a specific limit (between $8M_\odot$ and $11M_\odot$), although the precise value of this limiting mass is still unknown. Like all stars, These stars also begin their existence in the main sequence by burning hydrogen. After all of the fusible hydrogen in the core has been consumed, the star begins burning helium producing carbon and oxygen, which heats the overlying hydrogen shell, pushing the star's outer layers to expand dramatically. The star has reached supergiant status. The core continues to shrink and burn carbon, neon, oxygen, and silicon after fusing all helium. Each successive nuclear burning phase occurs in the layer outside of the previously active layers, creating an onion-like structure of various burning and inert shells [13]. A catastrophic scenario occurs when the star's core is entirely made of iron after the silicon burning phase. The iron nucleus cannot be further fused to produce thermonuclear energy since it is one of the most stable nuclei. An energy crisis arises in the core of these massive stars in their evolution. Gravitational contraction proceeds until the temperatures in the center regions are high enough for photons with sufficient energy to destroy iron nuclei in a photo-disintegration event. This event produces helium and neutrons, and with sufficient core temperature, even the helium is photo-disintegrated into protons and neutrons. The core continues to collapse, resulting in the fusion of protons and electrons to form more neutrons, which is called neutronization. Since neutrons are fermions, they obey Pauli's exclusion principle, and hence results in the neutron degeneracy pressure which opposes the further collapse. However, not all the iron is destroyed; the iron found outside the region where photo-disintegration occurs survives and covers the neutron-rich inner core, marking a neutron star's birth [13]. The core eventually rebounds when its density exceeds the typical nuclear density. This core bounce produces a shock wave propagating to the star's outer layers, resulting in an explosion of the star's outer layers. This process is called a Type-II Supernova. The neutron star becomes a black hole if the remnant's mass is approximately more than 3M$_\odot$. The upper limit for mass is also not precisely known and highly depends on the internal structure of the neutron star.

In the present calculations we make use of TOV equations as well as RMF model. The two methods are described in brief in the following sections.

3. Methodology

The heavenly objects like neutron star are relativistic objects, so the fundamental equations that describe the overall structure in equilibrium of a spherically symmetric, non-rotating star under the effect of gravity and general relativity are used to study the internal dynamics of the neutron star. These equations are known as Tolman-Oppenheimer-Volkoff (TOV) equations [13].

With the limitations we have due to the nature of neutron stars, our data collection would primarily be acquired through computer-generated theoretical models. Stellar structure equations formulate under the assumption that the neutron star is in hydrostatic equilibrium. Hydrostatic equilibrium constitutes being in equilibrium in which the internal pressure exerted by the star and the gravitational force exerted onto the star are equal that lead to the spherically symmetric shape [14]. Under these conditions, we have the following general structure equations for mass-radius relation and pressure-radius relation [15]. The equation for mass-radius relation is given by,

$$\frac{dm}{dr} = \frac{\left(4\,\pi\, r^2 \epsilon(r)\right)}{c^2}$$

Similarly, the equation that relates the variables pressure and radius is given by,

$$\frac{dp}{dr} = -\frac{\left(G\,\epsilon(r) m(r)\right)}{\left(c^2\, r^2\right)}$$

The Tolman-Oppenheimer-Volkoff (TOV) equation for pressure-radius is given by,

$$\frac{dp}{dr} = -\frac{\left(G\ \epsilon(r) m(r)\right)}{\left(c^2\, r^2\right)}\left[1 + \frac{p(r)}{\epsilon(r)}\right]\left[1 + \frac{\left(4\ \pi\ r^3 p(r)\right)}{m(r)\, c^2}\right]\left[1 - \frac{\left(2\ G\ m(r)\right)}{c^2\, r}\right]^{-1}$$

The Relativistic Mean Field (RMF) model used in the present calculations starts with the Lagrangian density that contains the

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contribution from σ-, ω- and ρ-mesons, δ-mesons up to the 2nd order, is given by,

$$
\begin{aligned}
\mathcal{L} &= \sum_{\alpha=n,p} \bar{\psi}_\alpha \left(\gamma^\mu \left(i\partial_\mu - g\omega\omega_\mu - \frac{1}{2} g\rho\tau_\alpha \cdot \rho_\mu\right) - (M - g\sigma\sigma) - g\delta\tau_\alpha \cdot \delta\right)\psi_\alpha \\
&+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\zeta_0}{4!}g^2\omega(\omega_\mu\omega^\mu)^2 - g\sigma m_\sigma^2 M\frac{k_3}{3!} + \frac{k_4}{4!}\frac{g\sigma}{M}\sigma^3 \\
&+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\frac{g\sigma\sigma}{M}\left(\eta_1 + \frac{\eta_2}{2}\frac{g\sigma\sigma}{M}\right)m_\omega^2\omega_\mu\omega^\mu \\
&+ \frac{1}{2}\eta_\rho m_\rho^2\frac{g\sigma\sigma}{M}(\rho_\mu \cdot \rho^\mu) + \frac{1}{2}m_\rho^2(\rho_\mu \cdot \rho^\mu) - \frac{1}{4}R_{\mu\nu}R^{\mu\nu} - \Lambda_\omega g^2\omega g^2\rho(\omega_\mu\omega^\mu)(\rho_\mu \cdot \rho^\mu) \\
&+ \frac{1}{2}\partial_\mu\delta\partial^\mu\delta - \frac{1}{2}m_\delta^2\delta^2
\end{aligned}
$$

The Euler-Lagrange equations of motion for the meson fields are obtained using the relativistic mean field approximation [21]. These equations are given by,

$$
\begin{aligned}
m_\sigma^2\sigma\sigma &= g\sigma\rho_s(r) - m_\sigma^2\frac{g\sigma}{M}\sigma^2 + \frac{k_3}{2} + \frac{k_4}{6}\frac{g\sigma\sigma}{M} + \frac{g\sigma}{2M}\left(\eta_1 + \eta_2\frac{g\sigma\sigma}{M}\right)m_\omega^2 \\
m_\omega^2\omega\omega &= g\omega\rho(r) - \frac{g\sigma}{M}\left(\eta_1 + \eta_2\frac{g\sigma\sigma}{M}\right)m_\omega^2 - \frac{1}{6}\zeta_0 g^2\omega\omega^3 - 2\Lambda_\omega(g\omega g\rho\rho)^2\omega \\
m_\rho^2\rho\rho &= \frac{1}{2}g\rho\rho^3(r) - \eta_\rho\frac{g\sigma\sigma}{M}m_\rho^2\rho - 2\Lambda_\omega(g\omega g\rho\omega)^2\rho \\
m_\delta^2\delta\delta &= g\delta\rho_s 3(r)
\end{aligned}
$$

Where the different variables in the above equations are given by,

$\rho_s(r) = \sum_{\alpha=n,p} <\bar{\psi}_\alpha\gamma^0\psi_\alpha> = \rho_{sn} + \rho_{sp}$, $\rho(r) = \sum_\alpha <\bar{\psi}_\alpha\psi_\alpha> = \rho_n + \rho_p$

$\rho 3(r) = \sum_\alpha <\bar{\psi}_\alpha\tau_3\psi_\alpha> = \rho_p - \rho_n$ $\rho_s 3(r) = \sum_\alpha <\bar{\psi}_\alpha\tau_3\gamma^0\psi_\alpha> = \rho_{sp} - \rho_{sn}$

$M_\alpha^*(\alpha = n, p)$ is the effective mass of nucleons given by the relation

$M_n^* = M - g\sigma\sigma + g\delta\delta,$ $M_p^* = M - g\sigma\sigma - g\delta\delta$

The expression for the energy density (E and) and pressure (P) are obtained from the given Lagrangian density using energy-momentum tensor as given below relation given by

$$T^{\mu\nu} = \sum_i \partial^\nu\phi_i\,\partial^\mu\phi_i - g^{\mu\nu}\mathcal{L}$$

,

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The energy density follows from the zeroth component and the pressure from the third component of the energy-momentum tensor. Their expressions are given as [22] :

$$
\begin{aligned}
E \; &= \frac{2}{(2\pi)^3}\sum_{\alpha=n,p}\int k_{\alpha 0} d^3k E_i^*(k) + \rho g\omega\omega + \frac{m_\sigma^2\sigma^2}{2} \\
&+ \frac{k_3}{3!}\frac{g\sigma\sigma}{M} + \frac{k_4}{4!}\frac{g^2\sigma\sigma^2}{M^2} - \frac{\zeta_0 g^2\omega\omega^4}{4!} - \frac{m_\omega^2\omega^2}{2}\left(1+\eta_1\frac{g\sigma\sigma}{M}+\frac{\eta_2}{2}\frac{g^2\sigma\sigma^2}{M^2}\right) \\
&+ \frac{\rho^3 g\rho\rho}{2} - \frac{1}{2}\left(1+\eta_\rho\frac{g\sigma\sigma}{M}\right)\frac{m_\rho^2\rho^2}{2} - \Lambda_\omega g^2\omega g^2\rho\omega^2\rho^2 + \frac{m_\delta^2\delta^2}{2} \\
P \; &= \frac{2}{3(2\pi)^3}\sum_{\alpha=n,p}\int k_{\alpha 0} d^3k k^2 E_i^*(k) - \frac{m_\sigma^2\sigma^2}{2} \\
&+ \frac{k_3}{3!}\frac{g\sigma\sigma}{M} + \frac{k_4}{4!}\frac{g^2\sigma\sigma^2}{M^2} + \frac{\zeta_0 g^2\omega\omega^4}{4!} + \frac{m_\omega^2\omega^2}{2}\left(1+\eta_1\frac{g\sigma\sigma}{M}+\frac{\eta_2}{2}\frac{g^2\sigma\sigma^2}{M^2}\right) \\
&+ \frac{\rho^3 g\rho\rho}{2} + \frac{1}{2}\left(1+\eta_\rho\frac{g\sigma\sigma}{M}\right)\frac{m_\rho^2\rho^2}{2} + \Lambda_\omega g^2\omega g^2\rho\omega^2\rho^2 - \frac{m_\delta^2\delta^2}{2}
\end{aligned}
$$

From the above equations we get the pressure as shown below,

$$
\mathrm{P} = g_\sigma\epsilon^2 + g_\omega\epsilon^3 + g_\rho\epsilon^4 - \frac{2}{1}g_2\epsilon^2 - g_3\epsilon^3 - \frac{4}{1}(g_\sigma - m_\sigma)m_\sigma^2\epsilon^4 - \frac{4}{1}(g_\omega - m_\omega)m_\omega^2\epsilon^4 - \frac{4}{1}(g_\rho - m_\rho)m_\rho^2\epsilon^4
$$

The results of the present calculations are shown below in the figures.

4. Result and Discussion

In the present work we have used the classical and relativistic mean field models to calculate mass and pressure with the distance within the neutron star. The variation of the pressure as a function of distance is shown in Fig. 1.

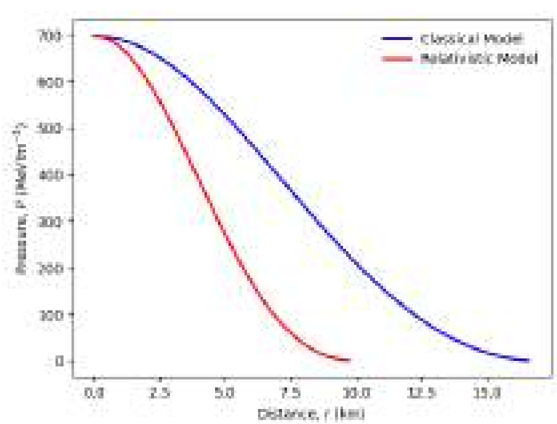

Fig. 1.: The variation of pressure with distance within neutron star.

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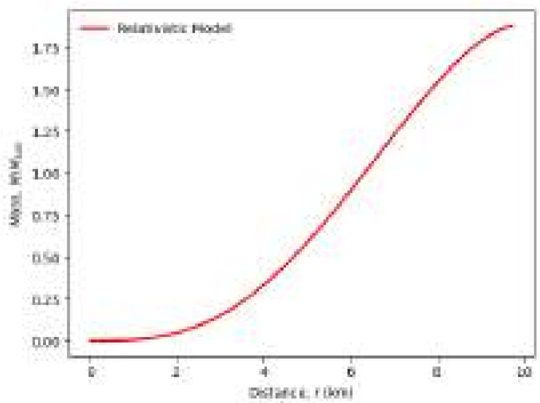

Fig. 2.: The variation of mass as a distance.

For a central density of 1665.3 MeV/fm^3, the simulation predicts a neutron star with a total mass of approximately 1.8765 times the mass of the Sun. The radius of the neutron star is calculated to be approximately 9.7167 km, providing insights into its compact structure. The variation of the mass with distance is shown in Fig. 2.

In Fig. 3 we present the comparison of the energy density as a function of pressure of the star for various parameters. In our calculations we find pressure and energy density are indeed related, but their relationship is more complex but not in a simple proportion.

The pressure inside a neutron star arises primarily from the degeneracy pressure and Pauli Exclusion Principle is obeyed that governs the behavior of fermions (such as neutrons, protons and electrons) in dense matter like neutron star. This pressure resists gravitational collapse and plays a crucial role in supporting the neutron star against its own gravity.

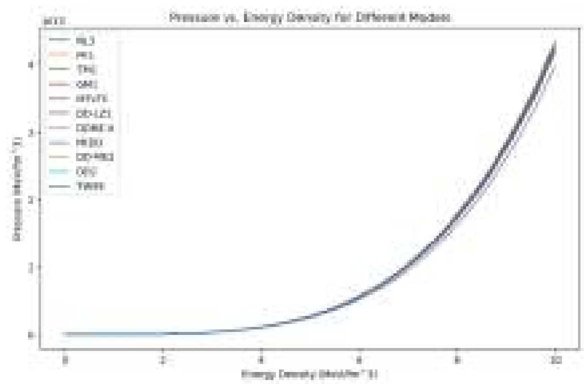

Fig. 3.: Comparison of Energy density of Neutron star with pressure using different models.

The energy density represents the total energy contained within a given volume of the neutron star. It includes contributions from the rest mass energy of particles (like neutrons and protons), kinetic energy, and potential energy arising from interactions among the particles. The relationship between pressure and energy density can vary significantly in the different regions of a neutron star due to the factors such as varying composition, density and temperature. For instance, in the core region the density is highest and the equation of state is stiff. The stiff equation of state means that a small change in density leads to large change in pressure. However, in the outer layers of the star the density is

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comparatively lower, so the equation of state is soft. A stiff equation of state typically implies that small changes in density results in large changes in pressure

5. Conclusion

In summary, our research has yielded significant insights into the properties and behavior of neutron stars through the development of a novel equation of state (EOS) derived from the Relativistic Mean Field (RMF) framework. By employing sophisticated theoretical models and computational simulations, we have made substantial progress in understanding the internal structure and dynamics of these enigmatic celestial bodies.

Our methodology involved the rigorous application of the RMF theory, which considers the interaction of nucleons through meson exchange, to derive an equation of state that captures the complex relationship between pressure, density, and energy density within neutron stars. Through numerical techniques and computational simulations, we have solved the Tolman-Oppenheimer-Volkoff (TOV) equation and employed the Runge-Kutta method to explore the mass-radius relation and pressure-density relation of neutron stars.

The results of our simulations have provided valuable insights into the properties of neutron stars. Specifically, for a central density of 1665.3 MeV/fm^3, our model predicts a neutron star with a total mass of approximately 1.8765 times the mass of the Sun and a radius of approximately 9.7167 km, consistent with observational data and theoretical predictions.

Furthermore, our analysis included a comprehensive comparison of our derived EOS with various RMF models, including NL3, PK1, TM1, GM1, MTVTC, DD-LZ1, DDME-X, PKDD, DD-ME2, DD2, and TW99. By examining the behavior of nuclear matter across different models, we have justified the robustness and reliability of our EOS within the broader context of RMF theory. Our EOS satisfies the general behavior observed in all RMF models, underscoring its validity and applicability in describing the properties of neutron stars.

In conclusion, our research represents an advancement in the study of neutron stars and the application of relativistic mean-field theory. By developing a comprehensive understanding of the internal structure and dynamics of neutron stars, we have laid the

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groundwork for further exploration and discovery in the field of astrophysics from this Eos. Our novel EOS provides a valuable tool for astronomers and physicists alike, offering insights into the fundamental nature of matter under extreme conditions and contributing to our broader understanding of the universe.

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