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Analytical Solutions of Bacteria Growth Model via Iman Transform

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Abstract: In this paper, we obtain the analytical solutions of Bacteria Growth Model through an Integral transformation, namely the Iman Transform. For demonstrating the usefulness of Iman Transform, we consider two numerical applications. The finding of these numerical applications shows that Iman Transform provides the analytical solution of bacteria growth model without doing complicated computational work. It has been shown that the Iman Transform is a practical, dependable, and simple technique for obtaining the solutions to the bacteria growth model.

Keywords: *Iman Transform, Inverse Iman Transform, Differential Equations, Bacteria Growth Model.*

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1. Introduction

Now a day, Integral transforms are the best convenient and easy mathematical process for finding advance problems solution arose in several fields like technology, science, social sciences, commerce, economics and engineering. Integral transforms provide exact solution of problem without lengthy calculations that is the vital feature of integral transforms. Due to this vital feature of the integral transforms various investigators are involved to this field and acquaint with many integral transforms. Differential equations are involved to examine the real-life problems; including Biological Growth, Tumour Growth, Heat, Carbon Dating, Compound Interest, Chemical Reaction Problem, Mixture Problem, Compartment Problem, Electric Circuit, Trajectory Problem and Vibrations [1]. Furthermost problems in these areas are modeled via ordinary linear differential equations and made more reasonable. There exist numerous mathematical and analytical methods in the literature for resolving differential equations of different types [2-8]. Afterward, integral transforms methodologies give precise solutions of the problems therefore various researchers are developing new integral transforms [9-17], these days.

The study of growth problem is one of the challenging problems in many areas. Growth problem can be usualy used in the field of sciences, social science and among other subjects. Various masses in the real-worlds growth at a quantity proportional to their size. Various integral transforms have been solved the population growth problems. As various investigators involved to presenting the new integral transforms at the same time and as well applying the transforms to various fields, various equations in different domain. Cooling law of Newton's problem was solved by Sanap and Patil [18], with the help of Kushare transform. Kumar *et al.* [19], proposed a new integral transform "Rishi Transform" and resolved the first kind linear Volterra integral equations using "Rishi Transform". Aggarwal [20], obtained the solution of the Bacteria Growth Model via Rishi Transform. Aggarwal and other scholars [21-30], studied the growth and decay models using various integral transformations such as Laplace transform, Mohand transform, Kamal transform, Aboodh transform, Mahgoub transform, Sadik transform, Elzaki transform, Shehu transform, Sumudu transform and Sawi transform. Aggarwal and other scholars [31-36], comparatively studied various integral transformations and Mohand transform and solved the system of ordinary differential equations using them. Aggarwal and others [37-43], gave the different integral transforms duality relations. Patil [44], have been used (Laplace and Shehu) transforms to gain the solution of chemical science problems. Deshmukh *et al*. [45], utilized Emad Sara transform to solve the problems related to population growth and decay. Recently, ,Patil *et al*. [46], used the Applications of Karry-Kalim-Adnan Transformation (KKAT) in Growth and Decay Problems. Dinesh Thakur and P.C. Thakur [47], Employing Upadhyaya Transform for finding the solution of linear second kind Volterra Integral equation. Almardy *et al*. [48], Solved the systems of ordinary differential equations with the help of Iman Transform Approach.

Mathematically, the equation of growth is a first order linear ordinary differential equation. Growth can be expressed as the first order derivative of amount of physical material $M(t)$ is directly proportional to quantity of physical material $M(t)$ at time t hour. The living growth such as growth of plant, growth of bacteria, growth of a species, growth of cell, growth of an organ etc. are governed by linear ordinary differential equation of first order as below[26-30]:

$$\frac{d\,M(t)}{dt}\ \alpha\ M(t)$$

Therefore, $\frac{d\,M(t)}{dt} = \xi\, M(t)$; over the initial condition $M(0) = M_0$ at time $t = 0$ **(**1)

where, $M(t)$ and M_0 are the quantity of physical material at any time t and $t = 0$, that is the exponential growth at rate proportional to its quantity material. ξ be the proportionality rate and the equation (1) is called the act of natural growth.

The main purpose of this paper is to determine the solution of the bacteria growth model using newly developed Iman Transform Technique and its efficiency to solve bacteria growth problem effectively.

2. Definition of Iman Transform and Its Properties [48]

Definition 2.1: For an exponential order function, the Iman Transform is defined as:

$$I = \left\{ f(t) : \exists\, K,\ \lambda_1, \lambda_2 > 0, \left| f(t) \right| < K e^{-v^2 t} \right\} \quad (2)$$

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where, K be the finite constant number, $f(t)$ be the function in the set I and λ_1, λ_2 may be finite or infinite number v − factor of t variable.

Definition 2.2: The kernel function of Iman Transform symbolized by I(.), written in the integral form as:

$$\left.\begin{aligned} I[f(t)] &= \frac{1}{v^2}\int_0^\infty \exp\left(-v^2 t\right) f(t)\,dt = B(v),\ t \ge 0,\ \lambda_1 < v < \lambda_2;\ \text{and} \\ f(t) &= I^{-1}[B(v)],\quad t \ge 0 \end{aligned}\right\} \tag{3}$$

Here, the inverse of Iman Transform is denoted by I^{-1}.

with the help of Iman Transform, we can easily solve the mathematical models in health sciences, environmental sciences and Biochemistry, containing ordinary linear differential equation of first order. The aim of this study is to show the applicability of this interesting transform and operator $B(v)$ defined by the integral equations.

2.3. Iman Transformation Linearity Property [48]:

If Iman transform of functions $f_1(t)$ and $f_2(t)$ are $B_1(v)$ and $B_2(v)$, respectively, then Iman transform of $[mf_1(t)+nf_2(t)]$ is given by $[mB_1(v)+nB_2(v)]$, where m and n are arbitrary constants.

Proof: Using (1), we get

$$I[f(t)] = \frac{1}{v^2}\int_0^\infty \exp(-v^2 t)\, f(t)\,dt$$

$$\Rightarrow I[mf_1(t)+nf_2(t)] = \frac{1}{v^2}\int_0^\infty [mf_1(t)+nf_2(t)]\exp(-v^2 t)\,dt$$

$$\Rightarrow I[mf_1(t)+nf_2(t)] = m\left[\frac{1}{v^2}\int_0^\infty f_1(t)\exp(-v^2 t)\,dt\right] + n\left[\frac{1}{v^2}\int_0^\infty f_2(t)\exp(-v^2 t)\,dt\right]$$

$$\Rightarrow I[mf_1(t)+nf_2(t)] = mI[f_1(t)] + nI[f_2(t)]$$

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$$\Rightarrow I[mf_1(t)+nf_2(t)] = mB_1(v)+nB_2(v) \quad (4)$$

2.4. Derived Properties of Iman Transform [48]

$$\left.\begin{array}{ll} \text{First derivative}: & I[f'(t)] = v^2 I(v) - \frac{1}{v^2} f(0) = I\left[\frac{df(t)}{dt}\right] \\ \text{Second derivative}: & I[f''(t)] = v^4 I(v) - f(0) - \frac{1}{v^2} f'(0) = I\left[\frac{d^2 f(t)}{dt^2}\right] \\ \text{nth derivative}: & I[f^n(t)] = v^n I(v) - \sum_{k=0}^{n-1} \frac{1}{v^{4-2n+2k}} f^k(0) = I\left[\frac{d^n f(t)}{dt^n}\right] \end{array}\right\} \quad (5)$$

2.5. Tabulated Values

Iman Transform and Inverse of Iman Transform of some function are as below by Almardy *et al.* [48].

2.5(a) Iman Transform of Some functions

$f(t)$	$I[f(t)]=B(v)$	$f(t)$	$I[f(t)]=B(v)$
$I(1)$	$=\frac{1}{v^4},$	$I(\exp(at))$	$=\frac{1}{v^4-av^2}$
$I(t)$	$=\frac{1}{v^6},$	$I(\exp(-at))$	$=\frac{1}{v^4+av^2}$
$I(t^n)$	$=\frac{m!}{v^{2m+4}},\quad m\in N,$		

2.5(b) Inverse of Iman Transform

$I^{-1}[B(v)]$	$f(t)$	$I^{-1}[B(v)]$	$f(t)$
$I^{-1}\left(\frac{1}{v^4}\right)$	$=1,$	$I^{-1}\left(\frac{1}{v^4-av^2}\right)$	$=\exp(at)$
$I^{-1}\left(\frac{1}{v^6}\right)$	$=t,$	$I^{-1}\left(\frac{1}{v^4+av^2}\right)$	$=\exp(-at)$
$I^{-1}\left(\frac{m!}{v^{2m+4}}\right)$	$=t^n,\quad m\in N,$		

3. Bacteria Growth Model, Technique of Iman Transfrom and its Applications

In this fragment, Iman Transform Technique have been applied to find the solution of the general form of bacteria growth model. Two application have been used to establish the effectiveness of Iman Transform Technique.

3.1. Bacteria Growth Model

Consider the Malthus model [26-30] for the significance of the growth of the bacteria present in a certain culture consulting to Malthus model, at which bacteria grow rate in a certain culture is proportional to the quantity of bacteria present at any time t. Generally bacteria growth problem expressed as the rate proportional to the amount of bacteria $M(t)$, subsequently time t hours in the first order form of differential equation,

Mathematically, bacteria growth model defined as below from the equation (1)

$\frac{d\,M(t)}{dt} = \xi\,M(t)$; through the condition $M(0) = M_0$ at time $t = 0$. (6)

The equation of bacteria growth (6) is a first order linear ordinary differential equation. Where, $M(t)$ and M_0 are the quantity of bacteria at any time t and time $t = 0$, which is the exponential growth at rate proportional to its quantity of bacteria. ξ be the proportionality rate and the equation (6) is called the act of natural bacteria growth.

3.2. Solution Of The Bacteria Growth Model Via Iman Transform Technique

Applying Iman Transform to equation (6) both the sides, we get

$$I\left[\frac{d\,M(t)}{dt}\right] = I[\xi\,M(t)] \tag{7}$$

Substituting the Iman Transform of the first derivative value from equation (5) in equation (7), we obtain

$$v^2\, I\{M(t)\} - \frac{M(0)}{v^2} = \xi . I\,\{M(t)\} \tag{8}$$

Using the condition that at a time $t = 0$, the quantity of bacteria be $M(0) = M_0$, in equation (8) and oversimplification, we obtain

$$I\{M(t)\} = \frac{M_0}{v^4 + \xi v^2} \tag{9}$$

Operating inverse Iman Transform jointly to the equation (9) and using the table–2.5(b), we obtain

$$M(t) = M_0\, I^{-1}\left(\frac{1}{v^4 + \xi v^2}\right)$$

$$M(t) = M_0\, e^{\xi t} \tag{10}$$

which is the required number of bacteria in a certain culture at time t. Iman Transform Technique is one of the type of integral transform that provides abundant suitability in solving first order differential equations and obtain solution accurately that is coinciding with the existing result obtain by using the other integral transform.

3.3. Numerical Applications

Application-3.3.1.

Bacteria in a certain culture increases at a rate proportional to the number present. If the number of bacteria increases from 1000 to 2000 in one hour, estimate the number of bacteria present in a certain culture at the end of 1.5 hours.

Above stated bacteria growth application, mathematically, be expressed at rate proportional to the number of bacteria present in a certain culture as below [20];

$\frac{d\,M(t)}{dt} = \xi\, M(t)$; through the initial condition $M(0) = M_0$ at $t = 0$ (11)

Here, constant of proportionality be denoted by ξ and the number of bacteria at time t and $t = 0$ be denoted by M and M_0.

Applying Iman Transform to equation (11) both the sides, we get

$$I\left[\frac{d\,M(t)}{dt}\right] = I[\xi\, M(t)] \quad (12)$$

Substituting the Iman Transform values of the first derivative from equation (5) in equation (12), we obtain

$$v^2\, I\{M(t)\} - \frac{M(0)}{v^2} = \xi . I\,\{M(t)\} \quad (13)$$

Using the condition that at a time $t = 0$, the quantity of bacteria be $M(0) = M_0$ $=1000$, in equation (13) and oversimplification, we obtain

$$v^2\, I\{M(t)\} - \frac{1000}{v^2} = \xi . I\,\{M(t)\} \quad (14)$$

After re-arranging, we obtain

$$I\{M(t)\} = \frac{1000}{v^4 - \xi\, v^2} \quad (15)$$

Operating Inverse Iman Transform jointly to the equation (15) and using table –2.5(b), we obtain

$$M(t) = 1000\ I^{-1}\left(\frac{1}{v^4 - \xi\, v^2}\right)$$

$$M(t) = 1000\, e^{\xi t} \quad (16)$$

Also, at time $t = 0$ and $t = 1$, the number of bacteria are $M(0) = M_0$ and $M(1) = 2000$;

Substituting these values in equation (17), we obtain

$$2000 = 1000\, e^{\xi}$$

$$\Rightarrow \quad e^{\xi} = 2$$

$$\Rightarrow \quad \xi = \ln(2) = 0.692$$

Hence, $\xi = 0.692$ (18)

Again, at time $t = 1.5$, the number of bacteria $M(1.5)$ present in a certain culture be obtain by substituting the values of t, ξ and M in the equation $M(t) = M_0\, e^{\xi t}$.

Therefore, $M(1.5) = 1000 e^{1.5(0.692)}$;

$$\Rightarrow \quad M(1.5) \approx 2827.80;$$

Hence, $M(1.5) \approx 2827.80$. (19)

which is the required number of bacteria present in a certain culture at the time t..

This obtain solution by using Iman Transform Technique is accurately coinciding with the existing result obtain by using the Rishi Transform [20].

Application-3.3.2.

Bacteria in a certain culture rises at a rate proportional to the quantity of bacteria presently living in a certain culture. If after two years, bacteria in a certain culture have doubled, and after three years' bacteria in a certain culture is 20,000, estimate the number of bacteria initially in a certain culture.

Above stated bacteria growth application, mathematically be expressed as the rate proportional to the number of bacteria as below [20];

$\frac{d\,M(t)}{dt} = \xi\, M(t)$; through the condition

$$M(0) = M_0 \text{ at } t = 0 \qquad (20)$$

Here, constant of proportionality be denoted by ξ and the number of bacteria at time t and $t = 0$ be denoted by M and M_0.

Applying Iman Transform to equation (20) both the sides, we get

$$I\left[\frac{d\,M(t)}{dt}\right] = I[\xi\, M(t)] \qquad (21)$$

Substituting the Iman Transform values of the first derivative from equation (5) in equation (21), we obtain

$$v^2\, I\{M(t)\} - \frac{M(0)}{v^2} = \xi . I\,\{M(t)\} \qquad (22)$$

Using the condition that at time $t = 0$, the quantity of bacteria be $M(0) = M_0$ in equation (22) and oversimplification, we obtain

$$v^2\, I\{M(t)\} - \frac{M_0}{v^2} = \xi . I\,\{M(t)\} \qquad (23)$$

After re-arranging, we obtain

$$I\{M(t)\} = \frac{M_0}{v^4 - \xi\, v^2} \qquad (24)$$

Operating Inverse Iman Transform jointly to the equation (24) and using the table –**2.5(b),** we obtain

$$M(t) = M_0\, I^{-1}\left(\frac{1}{v^4 - \xi\, v^2}\right)$$

$$M(t) = M_0\, e^{\xi t} \qquad (25)$$

Also, at the time $t = 2$, the number of bacteria be $M(2) = 2M_0$;

Substituting these values in equation (25), we obtain

$$2M_0 = M_0\, e^{2\xi}$$

$$\Rightarrow \quad e^{2\xi} = 2$$

$$\Rightarrow \quad \xi = 0.347$$

Hence, $\xi = 0.347$ (26)

Again, at the time $t = 3$, the number of bacteria be $M(3) = 20{,}000$; substituting the values of t, ξ and M in equation $M(t) = M_0\, e^{\xi t}$, we obtain

$$20{,}000 = M_0 e^{3(0.347)};$$

$$20{,}000 = M_0 (2.832);$$

Hence, $M_0 = 7062$. (27)

which is the required number of bacteria initially in a certain culture.

this obtain solution by using Iman Transform Technique is accurately coinciding with the existing result obtain by using the Rishi Transform [20].

4. Discussion And Conclusion

- In this work, we have successfully applied Iman Transform Technique for finding the solution of bacteria growth model.
- Applicability and competency of Iman Transform is demonstrated by giving two mathematical application of bacteria growth model.
- We observed that the result depict that the Iman transform is a very efficient integral transform for solving the application of bacteria growth model.
- Iman transform provides the analytical solution of the application of bacteria growth model without doing complicated calculation work as compared to other integral transform.
- In future, the suggested scheme can be applied for determining the solutions of Tumor growth model, radioactive substance decay model, model of chemical kinetic, traffic model, electric circuit model, compartment models, diabetes detection model and compound interest and heat conduction problems related to various different fields.

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